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Exploiting Network Topology Information to Mitigate Ambiguities in VMP Localization

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***Abstract*—We investigate an extension to the probabilistic model of a wireless sensor network (WSN) in a recently proposed variational message passing (VMP) localization algorithm. This extension exploits network topology information to mitigate ambiguities in WSN localization schemes. We derive VMP schemes for direct (1-step) localization and 2-step localization where knowledge on the positions of 2-step neighbours is taken into account. We evaluate the performance of the algorithms in connected WSNs via Monte Carlo simulations. The results show that utilizing position information from sensors with which there is no direct connection improves the position estimates for sensors directly connected to few neighbour sensors. The most significant improvement is observed for sensors of degree two i.e. with two 1-step neighbours. However, further investigations considering more realistic WSNs with decentralized and (partly) centralized settings are required to obtain more conclusive results on the real performance gain achievable by exploiting information on absent connections.**

I. Introduction

Localization algorithms that can be deployed in a wireless sensor network (WSN) are key to enabling location-aware services in the government, public and private sectors [1], [2]. Probabilistic sensor localization methods based on belief propagation have been described in e.g. [2]–[4].

In [5], the authors proposed a new probabilistic localization scheme based on variational message passing (VMP). The suggested algorithm performs well; but when sensor density is low, localization ambiguities arise due to the topology of the network. Resolving such ambiguities will potentially improve the performance of the VMP localization algorithm.

The basic principle in the ambiguity solutions published in [6] and [4] is to exploit not only measured inter-sensor distances but also information on network topology in the localization algorithm. This is done by modelling the conditional probability density function (pdf) of the absence of a connection between two sensors in addition to the conditional pdf of the presence of a link between sensors.

Probabilistic modeling of measured inter-sensor distances and inferred sensor positions allows for inference about sensor positions with both belief propagation and variational inference methods. Exploiting information on network topology, such as the absence of a network connection, can be considered merely as a sophistication of the probabilistic model that describes the network. Hence, the model including absent connections used in [6] and [4] yields a solution to the ambiguity problem experienced in our VMP localization algorithm simulations in [5].

In this paper we investigate the impact on the VMP localization algorithm in [5] when the model for absent network connections from [6] is included. We give a short review of the Gaussian VMP localization algorithm and conduct Monte Carlo simulations to verify and illustrate the effect of the proposed extension. Finally we present our concluding remarks.

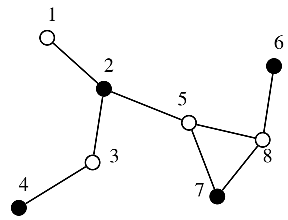

Figure 1. Example network with anchor sensors (black vertices), mobile sensors (white vertices) and their communication links (edges).

II. Models

Similarly to [6], we represent a network of wireless sensors scattered randomly and independently in the plane by a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with vertex set $\mathcal{V}$ and edge set $\mathcal{E}$ c.f. the example in Figure 1. Each $v \in \mathcal{V}$ is associated with a sensor and each $(r,t) \in \mathcal{E}$ represents a communication link between sensors r and t. In the following the term "sensor set" refers to $\mathcal{V}$. We subdivide the sensor set as $\mathcal{V} = \mathcal{V}_M \cup \mathcal{V}_A$, where $\mathcal{V}_M$ is the set of sensors at unknown positions and $\mathcal{V}_A$ is the set of sensors at known, fixed positions. The sensors in $\mathcal{V}_M$ and $\mathcal{V}_A$ are referred to as mobiles and anchors respectively.

Assume that with some probability $P_o(\boldsymbol{x}_r, \boldsymbol{x}_t)$ mobile sensor r obtains a noisy measurement $d_{r,t}$ of its distance from sensor t

$$d_{r,t} = \|\boldsymbol{x}_r - \boldsymbol{x}_t\| + w_{r,t} \tag{1}$$

where the vector $\boldsymbol{x}_r \in \mathbb{R}^2$ is the position of sensor r, $\|\cdot\|$ denotes the Euclidean norm and $w_{r,t}$ represents observation noise, which in this work is a zero-mean Gaussian random variable with variance $\sigma^2_{r,t}$.

Sensor r's prior knowledge of its position is assumed to be a circular symmetric Gaussian probability density function (pdf) $p_r(\boldsymbol{x}_r)$ in $\mathbb{R}^2$ with mean $\boldsymbol{\mu}_r = \mathrm{E}_{p_r}[\boldsymbol{x}_r]$ and component variance $\sigma_r^2 = \frac{1}{2}\mathrm{E}_{p_r}\left[\|\boldsymbol{x}_r - \boldsymbol{\mu}_r\|^2\right]$. Here $\mathrm{E}_{p_r}[\cdot]$ denotes expectation with respect to the pdf $p_r(\boldsymbol{x}_r)$. If $v \in \mathcal{V}_A$, then $\sigma_r^2 = 0$ and $p_r(\boldsymbol{x}_r)$ reduces to a Dirac's delta function localized at $\boldsymbol{\mu}_r$ in $\mathbb{R}^2$.

Similarly to [3], [4], [6], we denote the transmission range of a sensor by R and assume that the probability of observing neighbouring sensors falls off as

$$P_o(\boldsymbol{x}_r, \boldsymbol{x}_t) = \exp\left(-\frac{1}{2R^2}\|\boldsymbol{x}_r - \boldsymbol{x}_t\|^2\right). \tag{2}$$

Conditioned on the positions of the sensors involved, the individual distance measurements in the network are independent. This enables us to write the conditional pdf of a given distance observation as

$$p\left(d_{r,t} \,|\, \boldsymbol{x}_r, \boldsymbol{x}_t\right) = \begin{cases} P_o(\boldsymbol{x}_r, \boldsymbol{x}_t) p_w(d_{r,t} - \|\boldsymbol{x}_r - \boldsymbol{x}_t\|) & \text{if } d_{r,t} \in \mathbb{R}, \\ 1 - P_o(\boldsymbol{x}_r, \boldsymbol{x}_t) & \text{if } d_{r,t} = \text{nil}, \end{cases} \quad (3)$$

where $p_w(\cdot)$ is the observation noise pdf. To account for the case where no measurement is obtained we extend the definition domain of the distance measurements $d_{r,t}$ from $\mathbb{R}$ to $\mathbb{R} \cup \{\text{nil}\}$. Hence, "nil" denotes a non-existing measurement. From (3) it follows that position information is embedded in the knowledge of the absence of a connection between two sensors too. Provided that the sensors have access to the position of the sensors with which no distance measurement can be made, each sensor can exploit this information to resolve and/or improve its position estimate.

Given a sensor network, let $\mathcal{X} = \{\boldsymbol{x}_i : i \in \mathcal{V}_M\}$ denote the set of unknown sensor positions and $\mathcal{D} = \{d_{r,t} \,|\, (r,t) \in \mathcal{V}^2\}$ be the set of distance observations between all sensor pairs in the network. The joint pdf of distance observations and sensor positions reads

$$p\left(\mathcal{X}, \mathcal{D}\right) = p\left(\mathcal{D}|\mathcal{X}\right) p\left(\mathcal{X}\right) = \left(\prod_{(r,t) \in \mathcal{V}^2} p\left(d_{r,t} \,|\, \boldsymbol{x}_r, \boldsymbol{x}_t\right) \right) \left(\prod_{v \in \mathcal{V}_M} p_v\left(\boldsymbol{x}_v\right) \right). \quad (4)$$

This pdf is proportional to the posterior position pdf $p\left(\mathcal{X}|\mathcal{D}\right)$ which we use to estimate the position of any sensor $r \in \mathcal{V}_M$.

III. Message Passing for Localization

A. Factor Graph Representation of $p\left(\mathcal{X}, \mathcal{D}\right)$

The joint pdf in (4) can be represented by an undirected factor graph on which message passing methods can be applied to estimate the position posterior $p\left(\boldsymbol{x}_r|\mathcal{D}\right)$ of any mobile sensor in the network [7]–[9]. We create the graph by first defining the local factors

$$f_v(\boldsymbol{x}_v) = p_v\left(\boldsymbol{x}_v\right), \quad (5)$$

$$g_{r,t}(\boldsymbol{x}_r, \boldsymbol{x}_t) = p\left(d_{r,t} \,|\, \boldsymbol{x}_r, \boldsymbol{x}_t\right). \quad (6)$$

For each sensor $v \in \mathcal{V}$, we then draw a variable node, representing the sensor's position $\boldsymbol{x}_v$. We connect each $\boldsymbol{x}_v$, $v \in \mathcal{V}_M$ to a factor node $f_v(\boldsymbol{x}_v)$ representing the prior position pdf. For each pair of sensors $(r,t) \in \mathcal{V}^2$ we draw a factor node labelled $g_{r,t}(\boldsymbol{x}_r, \boldsymbol{x}_t)$ and connect the variable nodes $\boldsymbol{x}_r$ and $\boldsymbol{x}_t$ to it.

B. Gaussian Variational Message Passing for Localization

Variational methods aim at approximating a complex or intractable pdf by a simpler pdf [9], [10]. That is, using the notation in Section II, given the set $\mathcal{X}$ of unknown positions $\boldsymbol{x}_i, i \in \mathcal{V}_M$ and the set $\mathcal{D}$ of distance measurements, the posterior pdf $p\left(\mathcal{X}|\mathcal{D}\right)$ is approximated by a pdf that might belong to a certain family of pdfs satisfying certain constraints making their computation tractable. The selected pdf $q\left(\mathcal{X}\right)$ is an element in this family that minimizes the Kullback-Leibler divergence

$$\mathrm{KL}(q\left(\mathcal{X}\right) \,\|\, p\left(\mathcal{X}|\mathcal{D}\right)) = \int_{\mathcal{X}} q\left(\mathcal{X}\right) \ln \frac{q\left(\mathcal{X}\right)}{p\left(\mathcal{X}|\mathcal{D}\right)} \, d\mathcal{X}. \quad (7)$$

A well-known variational method is the mean field approximation where $q\left(\mathcal{X}\right)$ is assumed to factorize as $q\left(\mathcal{X}\right) = \prod_{\boldsymbol{x}_i \in \mathcal{X}} q_i\left(\boldsymbol{x}_i\right)$ [9]–[11]. This yields an iterative algorithm that approximates $p\left(\mathcal{X}|\mathcal{D}\right)$ by separately updating the factors $q_i\left(\boldsymbol{x}_i\right)$ in a sequential manner. Moreover, this algorithm can be interpreted as a message passing scheme. This interpretation is referred to as VMP [10]. For details on the VMP localization algorithm we refer the reader to [5].

For unconstrained VMP adapted to localization factor graphs a message from a variable node $\boldsymbol{x}_t$ to a neighbouring factor node $g_{r,t}(\boldsymbol{x}_r, \boldsymbol{x}_t) \in \mathcal{N}\left(\boldsymbol{x}_t\right)$ reads

$$m_{\boldsymbol{x}_t \to \mathcal{N}(\boldsymbol{x}_t)}(\boldsymbol{x}_t) = \prod_{h \in \mathcal{N}(\boldsymbol{x}_t)} m_{h \to \boldsymbol{x}_t}(\boldsymbol{x}_t). \quad (8)$$

Here $\mathcal{N}\left(\boldsymbol{x}_t\right)$ denotes the set of factor nodes neighbouring node $\boldsymbol{x}_t$. Messages from a factor node to variable node $\boldsymbol{x}_r$ are defined as follows:

$$m_{f_r \to \boldsymbol{x}_r}(\boldsymbol{x}_r) = p_r\left(\boldsymbol{x}_r\right), \quad (9)$$

$$m_{g_{r,t} \to \boldsymbol{x}_r}(\boldsymbol{x}_r) = \exp\left(\int_{\boldsymbol{x}_t} m_{\boldsymbol{x}_t \to g_{r,t}}(\boldsymbol{x}_t) \, \ln g_{r,t}(\boldsymbol{x}_r, \boldsymbol{x}_t) \, d\boldsymbol{x}_t \right). \quad (10)$$

The marginal update of the pdf estimate of $\boldsymbol{x}_r$ is computed as

$$q_r\left(\boldsymbol{x}_r\right) = \tfrac{1}{Z} m_{\boldsymbol{x}_r \to \mathcal{N}(\boldsymbol{x}_r)}(\boldsymbol{x}_r), \quad (11)$$

where Z is the normalization constant

$$Z = \int_{\boldsymbol{x}_r} m_{\boldsymbol{x}_r \to \mathcal{N}(\boldsymbol{x}_r)}(\boldsymbol{x}_r) \, d\boldsymbol{x}_r. \quad (12)$$

The unconstrained VMP messages described above may lead to unwieldy integral expressions in (10). This can be remedied by restricting the messages from variable nodes to factor nodes to be in the family $\mathcal{CN}$ of circular symmetric Gaussian pdfs, with mean $\hat{\boldsymbol{x}}_i$ and component variance $\hat{\sigma}_i^2$ for the ith node. As a result of this constraint, equations (8) and (11) must be modified according to (superscript G indicates Gaussian restriction)

$$m^G_{\boldsymbol{x}_r \to \mathcal{N}(\boldsymbol{x}_r)}(\boldsymbol{x}_r) = \underset{q'_r(\boldsymbol{x}_r) \in \mathcal{CN}}{\arg\min} \, \mathrm{KL}(q'_r\left(\boldsymbol{x}_r\right) \,\|\, \tilde{p}_r\left(\boldsymbol{x}_r\right)) \quad (13)$$

with

$$\tilde{p}_r\left(\boldsymbol{x}_r\right) = \frac{1}{Z} \prod_{h \in \mathcal{N}(\boldsymbol{x}_r)} m_{h \to \boldsymbol{x}_r}(\boldsymbol{x}_r) \quad (14)$$

and

$$q^G_r\left(\boldsymbol{x}_r\right) = m^G_{\boldsymbol{x}_r \to \mathcal{N}(\boldsymbol{x}_r)}(\boldsymbol{x}_r) \quad (15)$$

respectively. The solution to (13) is computed by finding the position and component variance estimates $\hat{\boldsymbol{x}}_r$ and $\hat{\sigma}_r^2$ of $q'_r\left(\boldsymbol{x}_r\right) \in \mathcal{CN}$ minimizing $\mathrm{KL}(q'_r\left(\boldsymbol{x}_r\right) \,\|\, \tilde{p}_r\left(\boldsymbol{x}_r\right))$. This can be done using numerical methods.

In [6], the authors investigated localizability of sensor networks in an idealized situation by Monte Carlo trials and found that most information relevant for computing the position of a sensor is local to that sensor. Furthermore, every uniquely determined network in the trials was also uniquely determined when the sensors only utilized information from the sensors with which they were connected and these sensors' connected neighbouring sensors (so-called "1-step" and "2-step" neighbours). That is, provided that network topology information is accessible sensor r can, in addition to its directly observable neighbours, identify sensors u for which $d_{r,t} \in \mathbb{R}^2$ and $d_{t,u} \in \mathbb{R}^2$ but $d_{r,u} = \text{nil}$. This can be achieved if each sensor in the network in addition to its position and variance estimates $\hat{\boldsymbol{x}}_r$ and $\hat{\sigma}_r^2$ broadcasts a list of its "1-step" neighbours' position estimates. A local approximation to (4) can be made by keeping only a subset of sufficiently informative edges. In accordance with [4], [6] we consider from now on algorithms that only rely on "1-step" and "2-step" neighbours in the network. The "1-step" and "2-step" neighbours of sensor r read

$$\mathcal{V}_{r,\text{1-step}} = \{t : (r,t) \in \mathcal{E}\}, \tag{16}$$

$$\mathcal{V}_{r,\text{2-step}} = \{t' : (t,t') \in \mathcal{E} \wedge t \in \mathcal{V}_{r,\text{1-step}}\}. \tag{17}$$

Replacing $\mathcal{V}^2$ in (14) by $\mathcal{V}_{r,\text{1-step}} \cup \mathcal{V}_{r,\text{2-step}}$ yields

$$\tilde{p}_r(\boldsymbol{x}_r) \propto m_{f_r \to \boldsymbol{x}_r}(\boldsymbol{x}_r) \prod_{t \in \mathcal{V}_{r,\text{1-step}} \cup \mathcal{V}_{r,\text{2-step}}} m_{g_{r,t} \to \boldsymbol{x}_r}(\boldsymbol{x}_r), \tag{18}$$

with $\propto$ denoting proportionality. It follows from (9) that

$$m_{f_r \to \boldsymbol{x}_r}(\boldsymbol{x}_r) \propto \exp\Big(-\tfrac{\|\boldsymbol{x}_r - \boldsymbol{\mu}_r\|^2}{2\sigma_r^2}\Big). \tag{19}$$

For $t \in \mathcal{V}_{r,\text{1-step}} \cup \mathcal{V}_{r,\text{2-step}}$, we have

$$m_{g_{r,t} \to \boldsymbol{x}_r}(\boldsymbol{x}_r) = \exp\Big(\mathrm{E}_{q_t^G}[\ln p(d_{r,t} \mid \boldsymbol{x}_r, \boldsymbol{x}_t)]\Big). \tag{20}$$

Inserting (19) and (20) in (18) we get the following expression for the KL divergence in (13):

$$\begin{aligned}
&\mathrm{KL}(q_r'(\boldsymbol{x}_r) \,\|\, \tilde{p}_r(\boldsymbol{x}_r)) \propto \\
&\ln\Big(\tfrac{\sigma_r^2}{\hat{\sigma}_r^2}\Big) - 1 + \tfrac{1}{2\sigma_r^2}\mathrm{E}_{q_r'}\Big[\|\boldsymbol{x}_r - \boldsymbol{\mu}_r\|^2\Big] \\
&+ \sum_{t \in \mathcal{V}_{r,\text{1-step}}} \Big(\tfrac{1}{2R^2}\mathrm{E}_{q_r'}\Big[\|\boldsymbol{x}_r - \boldsymbol{x}_t\|^2\Big] \\
&\qquad + \tfrac{1}{2\sigma_{r,t}^2}\Big(d_{r,t} - \mathrm{E}_{q_r'}\Big[\mathrm{E}_{q_t^G}\Big[\|\boldsymbol{x}_r - \boldsymbol{x}_t\|^2\Big]\Big]\Big)^2\Big) \\
&- \sum_{t' \in \mathcal{V}_{r,\text{2-step}}} \mathrm{E}_{q_r'}\Big[\mathrm{E}_{q_{t'}^G}\Big[\ln\Big(1 - \exp\Big(-\tfrac{1}{2R^2}\|\boldsymbol{x}_r - \boldsymbol{x}_{t'}\|^2\Big)\Big)\Big]\Big].
\end{aligned} \tag{21}$$

The right hand term in (21) depends implicitly on the mean and the variance of $q_r(\boldsymbol{x}_r)$. Hence, the mean $\hat{\boldsymbol{x}}_r$ and variance $\hat{\sigma}_r^2$ of $m^G_{\boldsymbol{x}_t \to \mathcal{N}(\boldsymbol{x}_r)}(\boldsymbol{x}_r)$ in (13) are the solutions minimizing this expression.

When only "1-step" neighbours are considered, the sum over $\mathcal{V}_{r,\text{2-step}}$ in (21) vanishes. We refer to the algorithm that only exploits this information from 1-step neighbours as as the direct VMP algorithm. It is similar to the proposed scheme in [5] with the modification that we have introduced $P_o(\boldsymbol{x}_r, \boldsymbol{x}_t)$. The algorithm that in addition also exploits the information from "2-step" neighbours is referred to as the 2-step VMP algorithm. Here, each sensor in the network broadcasts a list of its neighbours' position estimates in addition to its own position and variance estimates.

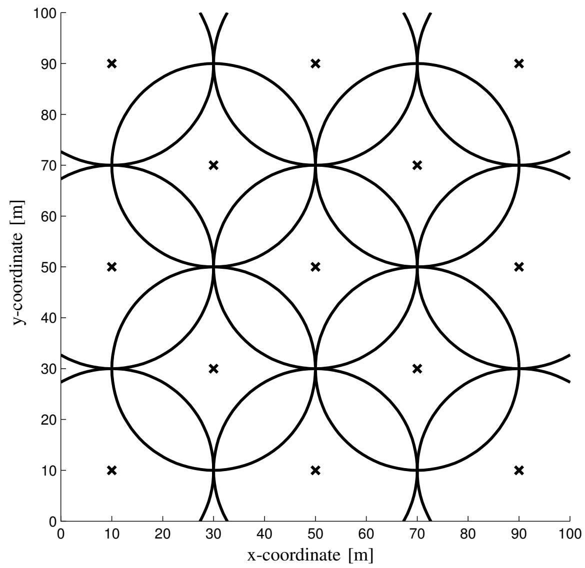

Figure 2. Simulation scenario: 13 anchor sensors and 10 static mobile sensors (not shown) in a connected network within $100\,\mathrm{m} \times 100\,\mathrm{m}$ in each Monte Carlo trial. $R = 20\,\mathrm{m}$ and $\sigma_{r,t}^2 = 0.01\,\mathrm{m}^2$. For simplicity, $d_{r,t} = d_{t,r}$.

IV. Numerical Experiments

We investigate by means of Monte Carlo simulations the potential performance gain that can be achieved using the 2-step VMP algorithm compared to the direct scheme. The considered scenario is depicted in Figure 2. In this scenario, the density of mobile sensors is low, and thus the WSN is with high probability sparse. Consequently, the probability that a mobile experiences a localization ambiguity (e.g. due to few network connections) is high. The low density of mobile sensors also leads to frequent occurrences of network realizations with groups of mobiles that are not connected directly or indirectly to any anchor. In such networks the detached mobile sensors can only estimate their positions relatively, irrespective of the applied localization method; the resulting large errors of the absolute position estimates drastically deteriorate the localization performance. As our focus is on alleviation of ambiguities we remedy these situations with detached sensors in the simulations by restricting the attention to connected networks, and discard networks with nodes that are not connected to the rest of the network. Doing so considerably restricts the validity of the results, but nonetheless allows for an assessment of the potential gain achievable by exploiting the information from 2-step neighbours.

In the simulations we assume that the anchors cannot communicate directly, i.e., they are not interconnected via a separate backbone network. For simplicity we further assume that $d_{r,t} = d_{t,r}$. In a Monte Carlo run the two algorithms each perform 20 iterations on the same WSN and measurement realization.

Figure 3 reports the estimated cumulative distribution function (cdf) of the absolute localization error. It appears that the

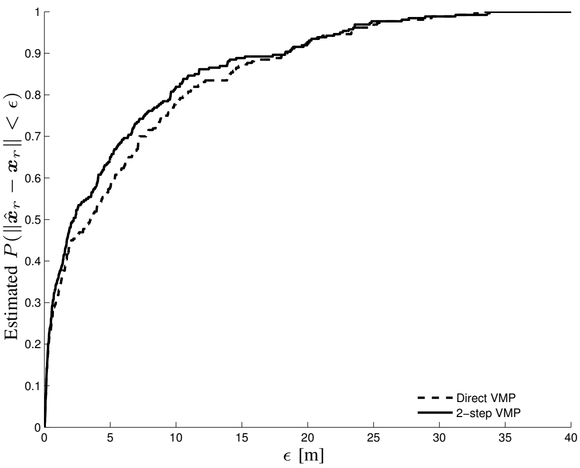

Figure 3. Estimated cdf of the absolute localization error for the direct (dashed line) and the 2-step (full line) VMP localization algorithms at iteration 20 based on 26 independent Monte Carlo trials.

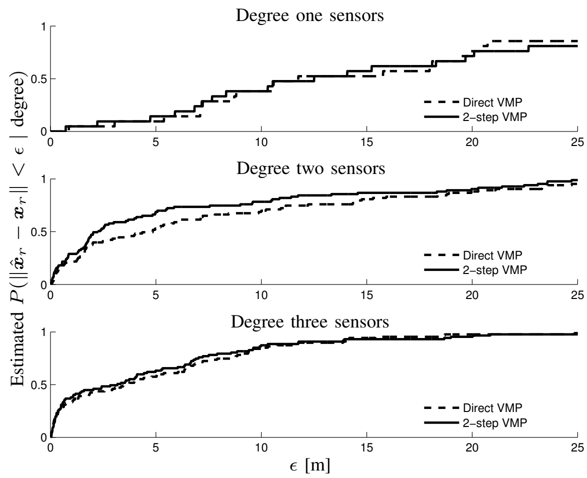

Figure 4. Estimated cdfs of the absolute localization error for sensors with degree one, two, and three for the direct (dashed line) and the 2-step (full line) VMP localization algorithms at iteration 20 based on 26 independent Monte Carlo trials.

Table I
RELATIVE FREQUENCIES OF SENSOR DEGREES IN THE SIMULATION.

Degree	1	2	3	4	≥ 5
Rel. freq. [%]	8	32	33	19	8

2-step VMP algorithm provides somewhat improved position estimates compared to the direct algorithm. To understand what leads to this improvement, we show in Figure 4 the estimated cdfs of the absolute localization error for sensors with degree one, two and three respectively, i.e. with one, two or three "1-step" neighbours. From the figure no significant difference between the two algorithms appears for sensors with degree one. One would anticipate an improvement in the 2-step VMP algorithm's favor. As Table I shows however, only 8% of the sensors have degree one. We conjecture that this small number of sensors along with the very low network density leads to networks in which degree one sensors in most cases are unable to exploit the "2-step" information from their neighbour. The estimated error cdf for degree 2 sensors obtained with the 2-step VMP algorithm is significantly higher than the cdf resulting from using the direct algorithm. This increase is due to the mitigation of ambiguities and hence the more precise localization of degree two sensors. The higher error cdf obtained with the 2-step VMP algorithm in Figure 3 stems mainly from these improved position estimates. For sensors with degree three we see a slight improvement in the 2-step VMP algorithm's favor. The cdfs for sensors with higher degrees (not depicted) show the same trend as the cdf for degree three sensors although with decreasing 2-step VMP improvement as the degree of the sensors increases.

V. CONCLUSION

We have investigated an ambiguity mitigating extension to the probability model used in our recently published Gaussian VMP localization algorithm. The simulation results show that the 2-step VMP algorithm improves the localization of degree two sensors. No significant improvement is seen for sensors with degree one. For sensors with higher degrees the performance improvement is slight in the tested scenario. Future work will include further investigations of more realistic WSN configurations with decentralized and (partly) centralized settings in order to obtain more conclusive results on the real performance gain achievable by exploiting information on absent connections between sensors.

VI. ACKNOWLEDGMENT

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REFERENCES

[1] N. Patwari, J. Ash, S. Kyperountas, I. Hero, A.O., R. Moses, and N. Correal, "Locating the nodes: Cooperative localization in wireless sensor networks," *Signal Processing Magazine, IEEE*, vol. 22, no. 4, pp. 54–69, July 2005.

[2] H. Wymeersch, J. Lien, and M. Win, "Cooperative localization in wireless networks," *Proceedings of the IEEE*, vol. 97, no. 2, pp. 427–450, February 2009.

[3] A. Ihler, I. Fisher, J.W., R. Moses, and A. Willsky, "Nonparametric belief propagation for self-localization of sensor networks," *Selected Areas in Communications, IEEE Journal on*, vol. 23, no. 4, pp. 809–819, April 2005.

[4] V. Savic and S. Zazo, "Sensor localization using generalized belief propagation in network with loops," in *17th European Signal Processing Conference (EUSIPCO 2009), Proceedings of the*, August 2009, pp. 75–79.

[5] C. Pedersen, T. Pedersen, and B. H. Fleury, "A variational message passing algorithm for sensor self-localization in wireless networks," in *Proceedings of the 2011 IEEE International Symposium on Information Theory*, August 2011, pp. 2064–2068.

[6] A. Ihler, I. Fisher, J.W., R. Moses, and A. Willsky, "Nonparametric belief propagation for self-calibration in sensor networks," in *Information Processing in Sensor Networks, 2004. IPSN 2004. Third International Symposium on*, April 2004, pp. 225–233.

[7] F. Kschischang, B. Frey, and H.-A. Loeliger, "Factor graphs and the sum-product algorithm," *Information Theory, IEEE Transactions on*, vol. 47, no. 2, pp. 498–519, February 2001.

[8] H.-A. Loeliger, "An introduction to factor graphs," *Signal Processing Magazine, IEEE*, vol. 21, no. 1, pp. 28–41, January 2004.

[9] T. Minka, "Divergence measures and message passing," Microsoft Research, Tech. Rep., 2005.

[10] J. Dauwels, "On variational message passing on factor graphs," in *Information Theory, 2007. ISIT 2007. IEEE International Symposium on*, June 2007, pp. 2546–2550.

[11] J. Winn and C. M. Bishop, "Variational message passing," *Journal of Machine Learning Research*, vol. 6, pp. 661–694, 2005.